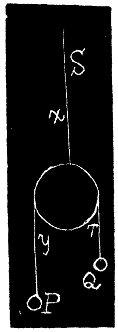
MECHANICS.

84. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Two weights P and Q are fastened by a weightless string that is strung over a single movable pulley. P is greater than Q. The weight of the pulley is $2R$. Find the tension of the string, (1) when the friction of the string on the pulley is neglected, (2) when it is considered.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

(1) Let S be the force acting upwards on the pulley, T the tension required, x, y as in the figure. The velocity of the pulley is dx/dt, the moving force $2R+2T-S$. The equation of motion, therefore, is



$$\frac{2R}{g}\cdot\frac{d^2x}{dt^2}=2R+2T-S\ldots\ldots(1).$$

Similarly for weights P and Q we have

$$\frac{P}{g}\left(\frac{d^2y}{dt^2}+\frac{d^2x}{dt^2}\right)=P-T\ldots\ldots(2).$$

$$\frac{Q}{g}\left(\frac{d^2y}{dt^2}-\frac{d^2x}{dt^2}\right)=T-Q\ldots\ldots(3).$$

Eliminating d^2y/dt^2 between (2) and (3) we get

$$\frac{d^2x}{dt^2}=\frac{2PQg-Tg(P+Q)}{2PQ}\ldots\ldots(4).$$

Eliminating d^2x/dt^2 between (1) and (4) we get

$$T=\frac{PQS}{2PQ+R(P+Q)}.$$

(2) In this case we will regard the pully as perfectly rough and disregard friction on the axle of the pulley. Three other cases are possible, as follows: Smooth axle, pulley imperfectly rough; rough axle, pully imperfectly rough: rough axle, pulley perfectly rough.

Let T'=tension caused by P, T''=tension caused by Q, θ=angle through which the pulley turns, a=radius of pulley, $k^2=\frac{1}{2}a^2$=radius of gyration.

Then we have

$$\frac{d^2x}{dt^2}=\frac{(2R+T'+T''-S)g}{2R}\ldots\ldots(5).$$

$$\frac{d^2y}{dt^2}+\frac{d^2x}{dt^2}=\frac{(P-T')g}{P}\ldots\ldots(6).\quad \frac{d^2y}{dt^2}-\frac{d^2x}{dt^2}=\frac{(T''-Q)g}{Q}\ldots\ldots(7).$$

$$\frac{2Rk^2}{g}\cdot\frac{d^2\theta}{dt^2}=a(T'-T'')\ldots\ldots(8).\quad a\theta=y\ \ldots\ldots(9).$$

From (9), $ad^2\theta/dt^2=d^2y/dt^2$. This in (8), with $k^2=\frac{1}{2}a^2$, gives

$$\frac{d^2y}{dt^2}=\frac{(T'-T'')g}{R}\ldots\ldots(10).$$

Eliminating d^2y/dt^2 and d^2x/dt^2 from (5), (6), (7), and (10), we get

$$(3P+2R)T'-PT''=PS.\quad QT'+(2R-3Q)T''=4RQ-QS.$$

$$\therefore\ T'=\frac{2PQ(R-S)+PRS}{P(3R-4Q)+R(2R-3Q)},\quad T''=\frac{2PQ(3R-S)+QR(4R-S)}{P(3R-4Q)+R(2R-3Q)}.$$

II. Solution by WILLIAM HOOVER, A. M., Ph. D., Professsor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let T, T' be the two tensions in the parts of the string to which P and Q are respectively attached, μ=the coefficient of friction between the string and pulley, a, k, the radius and radius of gyration, $m=2R/g$=the mass of the pulley, θ=the angle through which the pulley has rotated in the time t from the beginning of motion, s=the distance of the ascending weight above the earth at the time t.

For the motion of the weights, vertically,

$$\frac{P}{g}\frac{d^2s}{dt^2}=(P-T)\ldots\ldots(1),\quad \frac{Q}{g}\frac{d^2s}{dt^2}=(T'-Q)\ldots\ldots(2).$$

For the motion of the pulley,

$$mk^2\frac{d^2\theta}{dt^2}=a(T-T')\ldots\ldots(3).$$

Eliminating d^2s/dt^2 from (1) and (2), $PT'+QT=2PQ\ldots\ldots(4)$.
Again, from the theory of friction, $T'=Te^{-\mu\pi}=cT\ldots\ldots(5)$.
Substituting in (4),

$$T=\frac{2PQ}{cP+Q}\ldots\ldots(6),\quad T'=\frac{2cPQ}{cP+Q}\ \ldots\ldots(7).$$

(6) and (7) in (3) gives

$$mk^2\frac{d^2\theta}{dt^2}=\frac{2a(1-c)PQ}{cP+Q}\ldots\ldots(8).$$

Integrating (8), supposing $d\theta/dt=0$, when $t=0$,

$$mk^2\frac{d\theta}{dt}=\frac{2a(1-c)PQ}{cP+Q}t\ldots\ldots(9).$$

If there be no friction, as in (1) of the problem, $\mu=0$, $c=1$,

$$T=T'=\frac{2PQ}{P+Q}\ldots\ldots(10).$$

and (9) shows that there would be no rotation of the pulley.

85. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

A circular tube of radius a revolves uniformly about a vertical diameter with angular velocity $\sqrt{\frac{ng}{a}}$, and a particle is projected from its lowest point with such velocity that it can just reach the highest point; prove that the time of describing the first quadrant is $\sqrt{\frac{a}{(n+1)g}}\log(\sqrt{n+2}+\sqrt{n+1})$.

I. Solution by the PROPOSER.

Let $a\theta$ be the arc over which the particle has passed in any time t from the beginning of motion, R=reaction of the curve, g=the acceleration of gravity, and put $\sqrt{\frac{ng}{a}}=\omega$.

Resolving vertically and horizontally,

$$a\frac{d^2(\cos\theta)}{dt^2}=g-R\cos\theta\ldots\ldots(1),\quad a\frac{d^2(\sin\theta)}{dt^2}-\omega^2a\sin\theta=-R\sin\theta\ldots\ldots(2).$$

Eliminating R,

$$a\frac{d^2\theta}{dt^2}-a\omega^2\sin\theta\cos\theta=-g\sin\theta\ldots\ldots(3).$$

Integrating (3),

$$\frac{d^2\theta}{dt^2}=\frac{2g}{a}\cos\theta-\omega^2\cos^2\theta+C\ldots\ldots(4).$$

When $\theta=0$, $\frac{d\theta}{dt}=\frac{4g}{a}$; $\therefore$ $C=\frac{2g}{a}+\omega^2$, and (4) becomes